General Certificate of Education
June 2006
Advanced Level Examination

MATHEMATICS **MFP3**
Unit Further Pure 3

Monday 19 June 2006 9.00 am to 10.30 am

For this paper you must have:
- an 8-page answer book
- the **blue** AQA booklet of formulae and statistical tables

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MFP3.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.

Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 It is given that y satisfies the differential equation

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} - 5\frac{\mathrm{d}y}{\mathrm{d}x} + 4y = 8x - 10 - 10\cos 2x$$

(a) Show that $y = 2x + \sin 2x$ is a particular integral of the given differential equation. *(3 marks)*

(b) Find the general solution of the differential equation. *(4 marks)*

(c) Hence express y in terms of x, given that $y = 2$ and $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ when $x = 0$. *(4 marks)*

2 The function $y(x)$ satisfies the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{f}(x, y)$$

where $$\mathrm{f}(x, y) = \frac{x^2 + y^2}{xy}$$

and $$y(1) = 2$$

(a) Use the Euler formula

$$y_{r+1} = y_r + h\mathrm{f}(x_r, y_r)$$

with $h = 0.1$, to obtain an approximation to $y(1.1)$. *(3 marks)*

(b) Use the improved Euler formula

$$y_{r+1} = y_r + \tfrac{1}{2}(k_1 + k_2)$$

where $k_1 = h\mathrm{f}(x_r, y_r)$ and $k_2 = h\mathrm{f}(x_r + h, y_r + k_1)$ and $h = 0.1$, to obtain an approximation to $y(1.1)$, giving your answer to four decimal places. *(6 marks)*

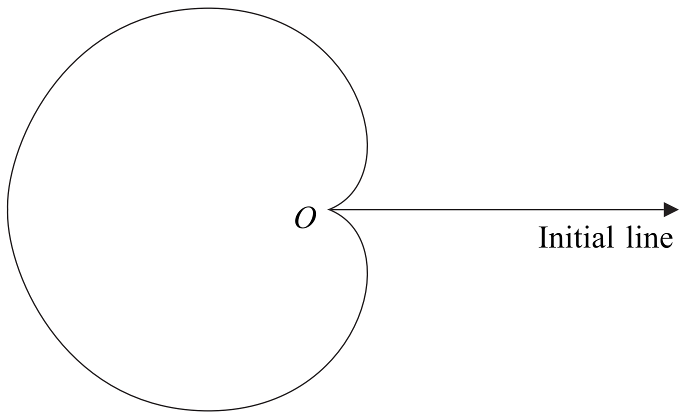
3 (a) Show that $\sin x$ is an integrating factor for the differential equation

$$\frac{dy}{dx} + (\cot x)y = 2\cos x$$ *(3 marks)*

(b) Solve this differential equation, given that $y = 2$ when $x = \frac{\pi}{2}$. *(6 marks)*

4 The diagram shows the curve C with polar equation

$$r = 6(1 - \cos\theta), \qquad 0 \leqslant \theta < 2\pi$$



(a) Find the area of the region bounded by the curve C. *(6 marks)*

(b) The circle with cartesian equation $x^2 + y^2 = 9$ intersects the curve C at the points A and B.

(i) Find the polar coordinates of A and B. *(4 marks)*

(ii) Find, in surd form, the length of AB. *(2 marks)*

5 (a) Show that $\lim_{a \to \infty} \left(\frac{3a+2}{2a+3}\right) = \frac{3}{2}$. *(2 marks)*

(b) Evaluate $\int_1^{\infty} \left(\frac{3}{3x+2} - \frac{2}{2x+3}\right) dx$, giving your answer in the form $\ln k$, where k is a rational number. *(5 marks)*

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6 (a) Show that the substitution

$$u = \frac{\mathrm{d}y}{\mathrm{d}x} + 2y$$

transforms the differential equation

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + 4\frac{\mathrm{d}y}{\mathrm{d}x} + 4y = \mathrm{e}^{-2x}$$

into

$$\frac{\mathrm{d}u}{\mathrm{d}x} + 2u = \mathrm{e}^{-2x}$$

(4 marks)

(b) By using an integrating factor, or otherwise, find the general solution of

$$\frac{\mathrm{d}u}{\mathrm{d}x} + 2u = \mathrm{e}^{-2x}$$

giving your answer in the form $u = \mathrm{f}(x)$. *(5 marks)*

(c) Hence find the general solution of the differential equation

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + 4\frac{\mathrm{d}y}{\mathrm{d}x} + 4y = \mathrm{e}^{-2x}$$

giving your answer in the form $y = \mathrm{g}(x)$. *(5 marks)*

7 (a) (i) Write down the first three terms of the binomial expansion of $(1 + y)^{-1}$, in ascending powers of y. *(1 mark)*

(ii) By using the expansion

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \ldots$$

and your answer to part (a)(i), or otherwise, show that the first three non-zero terms in the expansion, in ascending powers of x, of $\sec x$ are

$$1 + \frac{x^2}{2} + \frac{5x^4}{24}$$

(5 marks)

(b) By using Maclaurin's theorem, or otherwise, show that the first two non-zero terms in the expansion, in ascending powers of x, of $\tan x$ are

$$x + \frac{x^3}{3}$$

(3 marks)

(c) Hence find $\lim\limits_{x \to 0} \left(\dfrac{x \tan 2x}{\sec x - 1} \right)$. *(4 marks)*

END OF QUESTIONS

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